SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

November 2011

Assessment Task 1
Year 11

Mathematics Extension

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question if full marks are to be awarded.
- Answer in simplest exact form unless otherwise instructed.

Total Marks – 65

- Attempt sections A – D.
- Start each **NEW** section in a separate answer booklet.
- Hand in your answers in 4 separate bundles:
  Section A
  Section B
  Section C
  Section D

Examiner: *J. Chen*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x + \sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x + \sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

START A NEW ANSWER BOOKLET

SECTION A [15 marks] **Marks**

For these 10 questions there is one correct answer per question. Write down in your answer booklet the question number and letter of your answer.

1. **[1]**

$$\int \frac{1}{(1-2x)^3}\,.\,dx$$

(a) $-\frac{1}{4(1-2x)^2}+C$

(b) $\frac{1}{4(1-2x)^2}+C$

(c) $-\frac{1}{2(1-2x)^2}+C$

(d) $\frac{1}{2(1-2x)^2}+C$

2. Find the acute angle between the lines $x+3y=0$ and $x-2y=1$. **[1]**

(a) $135°$

(b) $45°$

(c) $22.5°$

(d) None of the above

3. Solve $\cos\left(x-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$, for $0 \le x \le 2\pi$. **[1]**

(a) $x=0,\ \frac{\pi}{3},\ 2\pi$

(b) $x=\frac{\pi}{6},\ \frac{11\pi}{6}$

(c) $x=\frac{\pi}{3},\ 2\pi$

(d) $x=\pi,\ \frac{4\pi}{3}$

4. P and Q are the points $(-4,3)$ and $(2,1)$ respectively. What are the coordinates of the point M which divides QP externally in the ratio $4:5$. **[1]**

(a) $M(26,-7)$

(b) $M(31,-3)$

(c) $M\left(-\frac{2}{3},\frac{17}{9}\right)$

(d) $M(-28,11)$

5. What is the exact value of $\tan 15°$? **[1]**

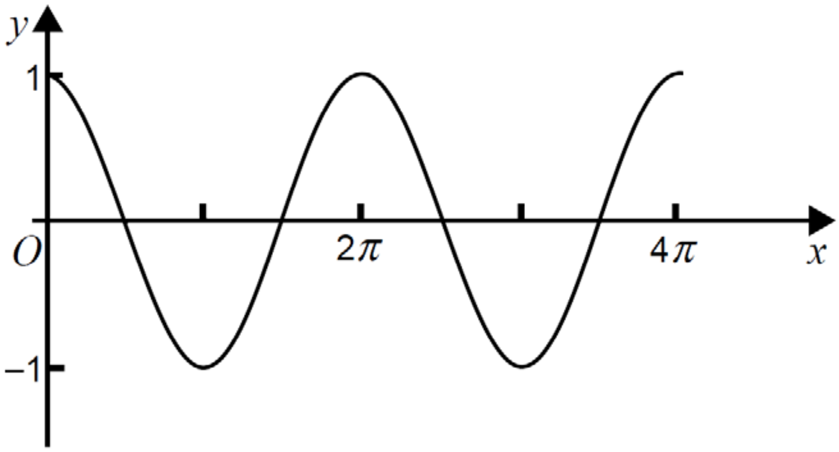
(a) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

(b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(c) $\frac{1-\sqrt{3}}{1+\sqrt{3}}$

(d) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

6. Part of the graph of a trigonometric function is shown below. **[1]**



A possible equation for the function is

(a) $y = \cos(2x)$

(b) $y = \cos x$

(c) $y = \sin(2x)$

(d) $y = \sin x$

7. The linear factors of $x^4 + x^3 - 3x^2 - 3x$ are **[1]**

(a) $x,\ x+\sqrt{3},\ x-\sqrt{3}$

(b) $x+1,\ x+\sqrt{3},\ x-\sqrt{3}$

(c) $x,\ x+1$

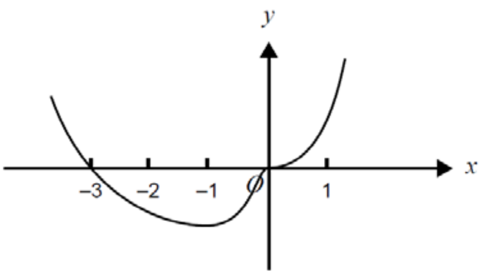
(d) $x,\ x+1,\ x+\sqrt{3},\ x-\sqrt{3}$

8. A continuous function $f(x)$ has the following properties. **[1]**

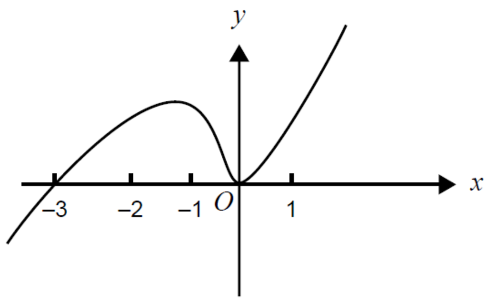
$$f(0) = 0$$
$$f(-3) = 0$$
$$f'(0) = 0$$
$$f'(-1) = 0$$
$$f'(x) > 0 \text{ for } x < -1$$
$$f'(x) < 0 \text{ for } x > -1,\ x \neq 0$$

Which one of the following could represent the graph of $f(x)$?

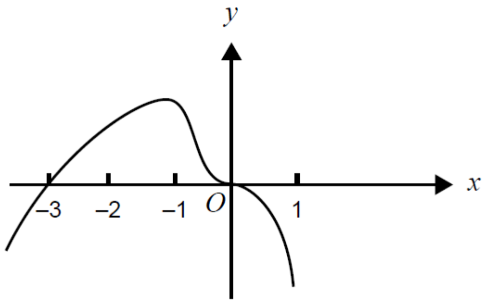
(a)



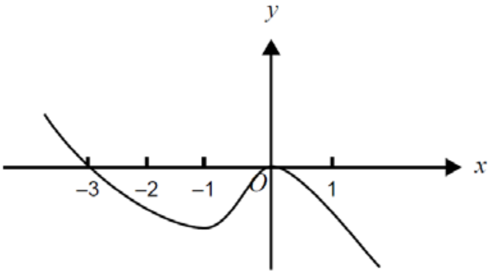
(b)



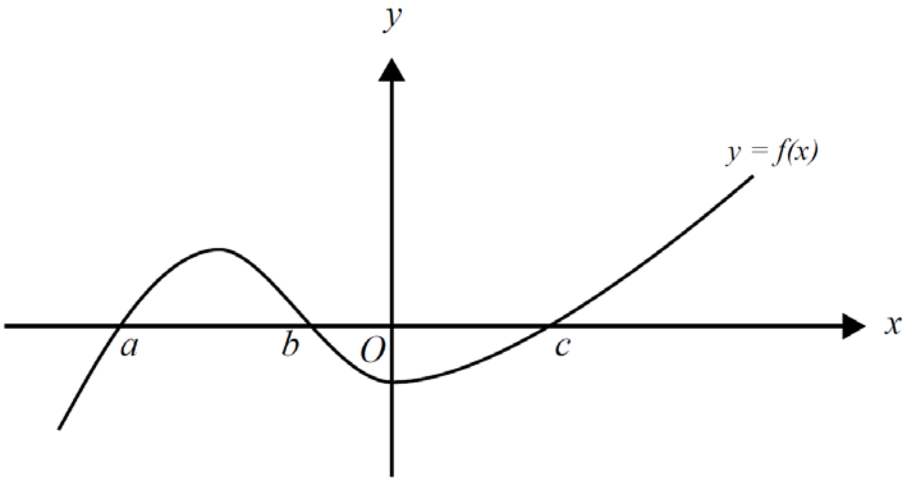
(c)



(d)



9. The diagram below shows part of the graph of a function $y = f(x)$. **[1]**



The total area bounded by the function $y = f(x)$ and the x-axis between $x = a$ and $x = c$ is given by

(a)
$$\int_a^c f(x)\,.\,dx$$

(b)
$$\int_a^b f(x)\,.\,dx + \int_c^b f(x)\,.\,dx$$

(c)
$$\int_a^b f(x)\,.\,dx + \int_b^c f(x)\,.\,dx$$

(d)
$$\int_b^c f(x)\,.\,dx - \int_a^b f(x)\,.\,dx$$

10. If $\int_0^4 f(x)\,.\,dx = 3$, then $\int_0^4 (3f(x) - 2)\,.\,dx$ is equal to **[1]**

(a) 9

(b) 1

(c) 7

(d) 3

End of Multiple Choice Section

11. Find all angles θ for which $\tan\theta = \sqrt{3}$. **[1]**

12. Sketch the graph of a function $y = f(x)$ such that $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$ for $0 \leq x \leq 2$. **[2]**

13. Prove that **[2]**

$$(\cot\theta + \operatorname{cosec}\theta)^2 = \frac{1 + \cos\theta}{1 - \cos\theta}$$

End of Section A

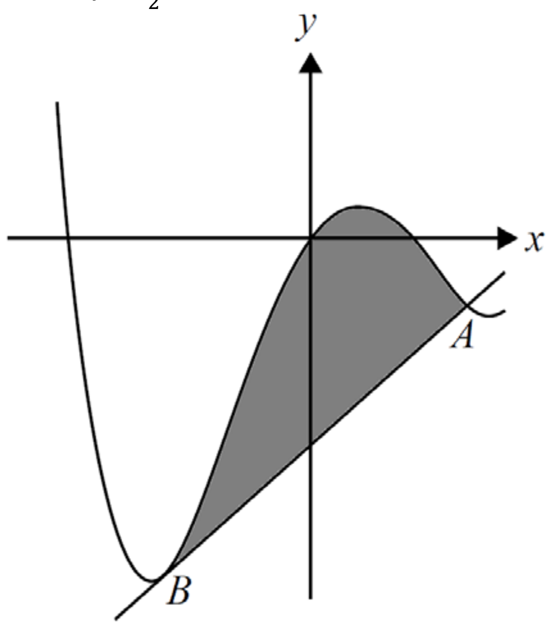
START A NEW ANSWER BOOKLET

SECTION B [20 marks] **Marks**

1. Solve $\cos\theta = \frac{1}{\sqrt{2}}$, for $0 \leq \theta \leq 2\pi$. **[2]**

2. The diagram below shows the curve with equation

$$y = \frac{1}{2}(2x^4 - x^3 - 5x^2 + 3x).$$



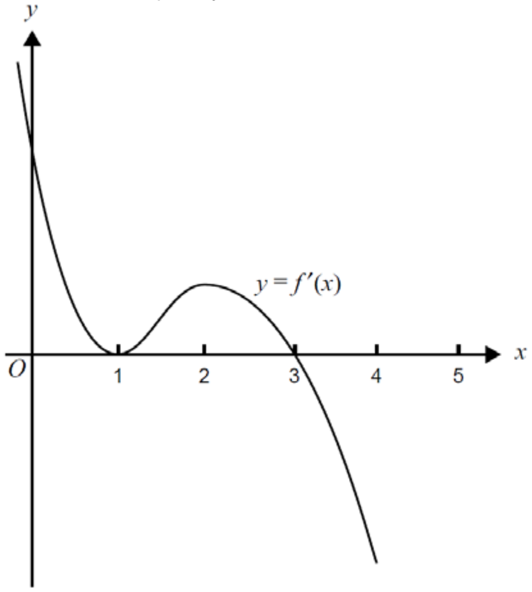
The normal to the curve at A where $x = 1$ is a tangent to the curve at B.

(i) Find the normal to the curve at A. **[1]**

(ii) Use the factor theorem, or otherwise, find the coordinates of B. **[3]**

(iii) Hence, find the area of the shaded region. **[2]**

3. The diagram below shows the derivative of a function $y = f(x)$. Sketch a possible graph of the function $y = f(x)$. **[2]**

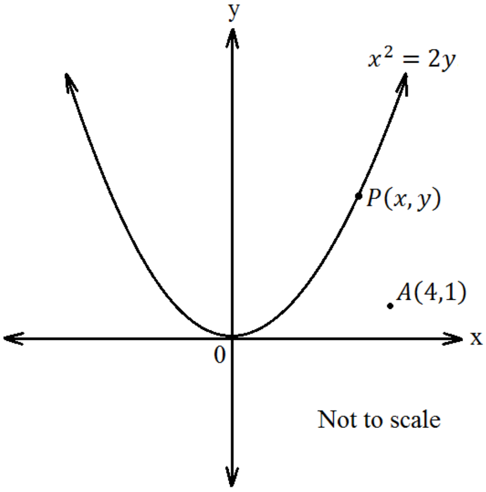


4.

(i) Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **[2]**

(ii) Hence, or otherwise, solve $\cos 3\theta + \sin 2\theta = \cos\theta$ for $0 \leq \theta \leq 2\pi$. **[3]**

5. The diagram below shows the graph of the parabola $x^2 = 2y$. The point $A(4,1)$ is outside the parabola while the point $P(x, y)$ is on the parabola as shown in the below diagram.



(i) If D is the distance between the two points A and P, show that **[1]**

$$D^2 = \left(\frac{1}{2}x^2 - 1\right)^2 + (x-4)^2$$

(ii) Find the value of x that produces the minimum value for D in the equation in part (i). **[3]**

(iii) Determine the minimum distance between A and P in exact form. **[1]**

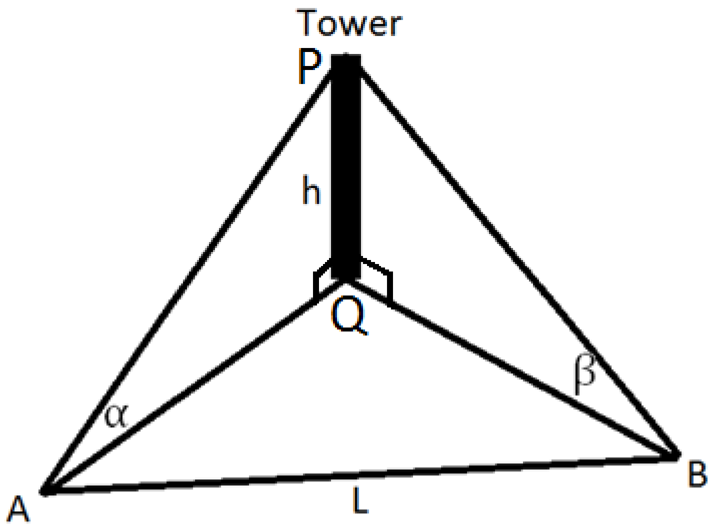
End of Section B

START A NEW ANSWER BOOKLET

SECTION C [16 marks] **Marks**

1. If $x = c$ is a double zero of $P(x)$ i.e. $P(x) = (x - c)^2 Q(x)$, show that $x = c$ is a zero of $P'(x)$. **[2]**

2. A tower has a height of h. At point A, the angle of elevation of the top of the tower is α, and at point B, the angle of elevation to the top of the tower is β. AB is separated by a distance of L and $\angle AQB = 60°$.



   (i) Write an expression for h in terms of α, β and L. **[3]**

   (ii) If $\tan\alpha \tan\beta = x$ and $(\tan\alpha + \tan\beta)^2 = 3x + x^2$, show that $h = L$. **[2]**

3.

   (i) Express $\sqrt{3}\cos\theta - \sin\theta$ in the form $\mathrm{R}\cos(\theta - \alpha)$, where $\mathrm{R} > 0$ and α is an angle in radians. **[2]**

   (ii) Hence, solve $\sqrt{3}\cos\theta - \sin\theta = 1$ for $0 \leq \theta \leq 2\pi$. **[2]**

4. $P(2p, p^2)$ and $Q(2q, q^2)$ are points on the parabola $x^2 = 4y$. The chord PQ always passes through the point $R(4, 0)$ when produced.

   (i) Given the equation of chord PQ is $y = \frac{1}{2}(p + q)x - pq$, show that $pq = 2(p + q)$. **[1]**

   (ii) M is the midpoint of chord PQ. Find the coordinates of M. **[1]**

   (iii) Describe the locus of M, indicating any restrictions on x value. **[3]**

End of Section C

START A NEW ANSWER BOOKLET

SECTION D [14 marks] **Marks**

1. Solve $\sin 2\theta - 3\sin^2\theta + 2\sin\theta = 0$ for $-\pi \le \theta \le \pi$. **[3]**

2. Solve for x, **[3]**

$$\frac{x}{1-x^2} \le 0$$

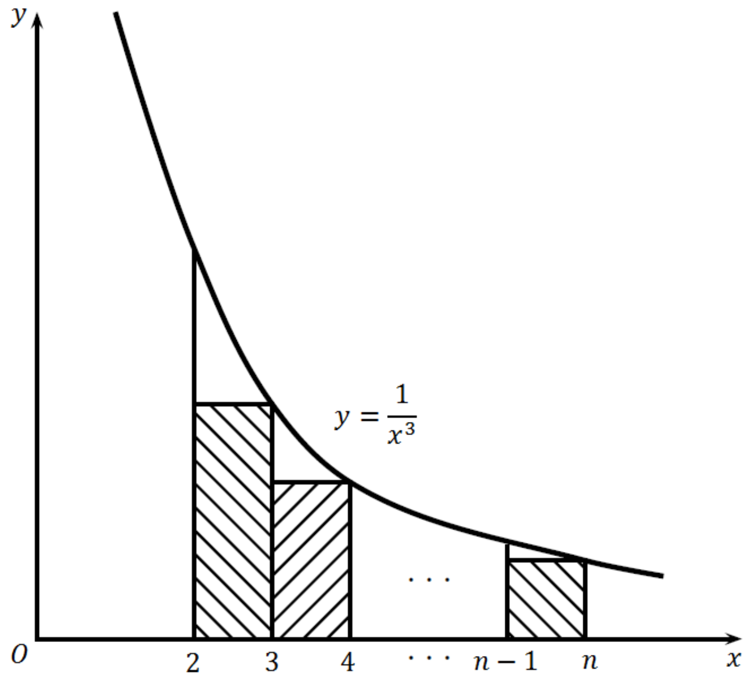
3. Let α and β be the solutions of **[4]**

$$x^2 - (a+d)x + ad - bc = 0$$

Prove that α^3 and β^3 are the solutions of

$$x^2 - (a^3 + d^3 + 3abc + 3bcd)x + (ad - bc)^3 = 0$$

4. The diagram shows the curve $y = \frac{1}{x^3}$, for $x > 0$.



(i) Find an expression for the sum of areas of the shaded rectangles between $x = 2$ and $x = n$, where n is a positive integer. **[1]**

(ii) Hence, or otherwise, prove that for any positive integer n, **[3]**

$$1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots + \frac{1}{n^3} < \frac{5}{4}$$

End of Section D
End of Exam

SECTION A

1. B

2. B

3. A

4. A

5. A

6. B

7. D

8. C

9. B

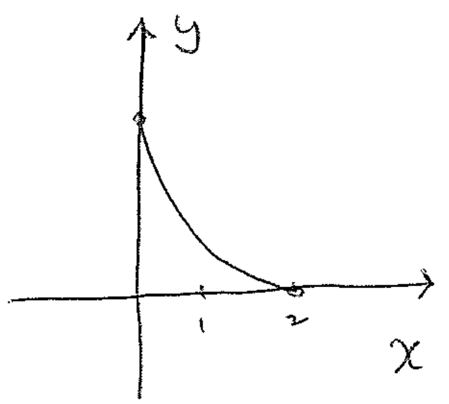
10. ~~C~~ B

11. $\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3} \pm n\pi$

$(n \in J^+)$ [1]

12.



[2]

13. $LHS = (\cot\theta + \operatorname{cosec}\theta)^2$

$= \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)^2$

$= \left(\frac{\cos\theta + 1}{\sin\theta}\right)^2$

$= \frac{(\cos\theta + 1)^2}{\sin^2\theta}$

$= \frac{(\cos\theta + 1)^2}{1 - \cos^2\theta}$

$= \frac{1+\cos\theta}{1-\cos\theta}$

$= RHS$

OR $RHS = \frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$= \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$

$= \frac{1 + 2\cos\theta + \cos^2\theta}{\sin^2\theta}$

$= \operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta\cot\theta + \cot^2\theta$

$= (\operatorname{cosec}\theta + \cot\theta)^2$

$= LHS$

[2]

Section B – Solutions

1. $\cos\theta = \frac{1}{\sqrt{2}} \quad 0 \le \theta \le 2\pi$

$\theta = \frac{\pi}{4}$ ✓ or $\frac{7\pi}{4}$ ✓ (2)

2 (i) $y = \frac{1}{2}(2x^4 - x^3 - 5x^2 + 3x)$

$y' = \frac{1}{2}(8x^3 - 3x^2 - 10x + 3)$

At $x = 1$, $y = -\frac{1}{2} \Rightarrow A = (1, -\frac{1}{2})$

At $x = 1$, $y' = -1$ = gradient of tangent.

$\therefore$ gradient of normal $= 1$.

Eqn of normal at A: $y + \frac{1}{2} = 1(x-1)$

$y = x - \frac{3}{2}$ ✓ (1)

(ii) $y = \frac{1}{2}(2x^4 - x^3 - 5x^2 + 3x)$ ①

$y = x - \frac{3}{2}$ ②

Sub in in ② $\Rightarrow x - \frac{3}{2} = \frac{1}{2}(2x^4 - x^3 - 5x^2 + 3x)$

$2x^4 - x^3 - 5x^2 + x + 3 = 0$ ✓

Then $(x-1)(x+1)$ is a factor $\Rightarrow x^2 - 1$ is a factor.

$$\begin{array}{r} 2x^2 - x - 3 \\ x^2 - 1 \overline{)\, 2x^4 - x^3 - 5x^2 + x + 3} \\ \underline{2x^4 - 0x^3 - 2x^2} \\ -x^3 - 3x^2 + x \\ \underline{-x^3 + 0x^2 + x} \\ -3x^2 \quad + 3 \\ \underline{-3x^2 \quad + 3} \\ 0 \end{array}$$ ✓

$\Rightarrow P(x) = (x-1)(x+1)(2x^2 - x - 3) = 0$

$= (x-1)(x+1)^2(2x-3) = 0$

2(ii) cont. Only one soln to left of A $\Rightarrow x = -1$

$\therefore B = \left(-1, \frac{-5}{2}\right)$ ✓

(3)

(iii) Area $= \int_{-1}^{1} \frac{1}{2}\left(2x^4 - x^3 - 5x^2 + 3x\right) - \left(x - \frac{3}{2}\right) dx$

$= \int_{-1}^{1} \left(x^4 - \frac{1}{2}x^3 - \frac{5}{2}x^2 + \frac{1}{2}x + \frac{3}{2}\right) dx$ ✓

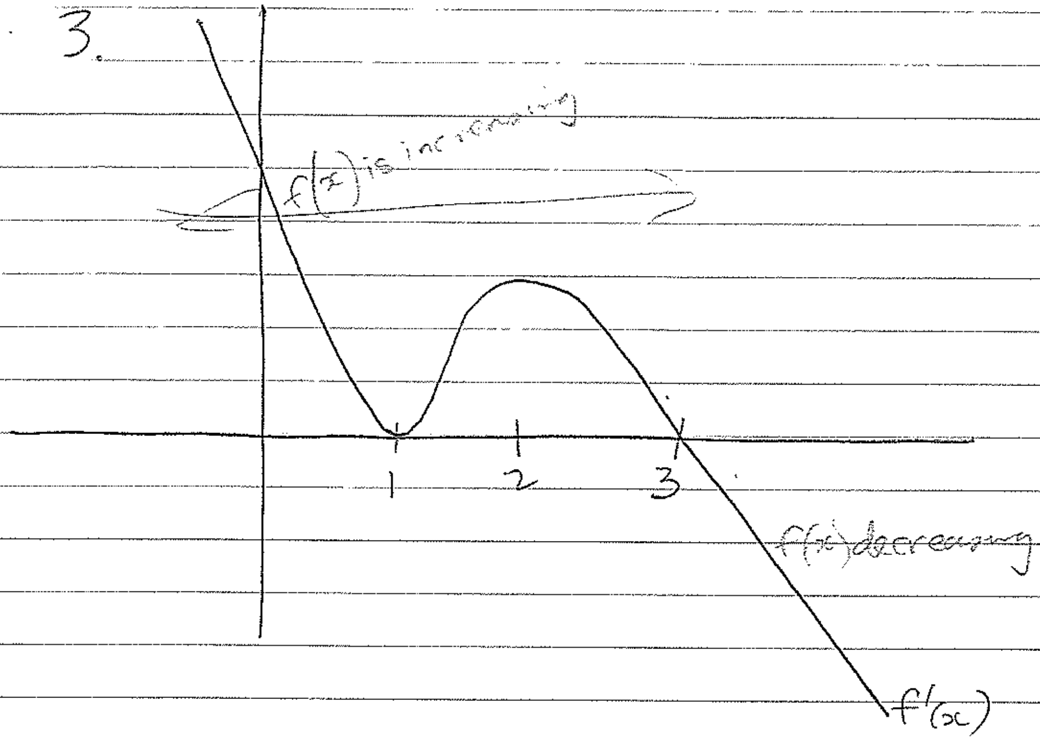
$= \left[\frac{x^5}{5} - \frac{x^4}{8} - \frac{5x^3}{6} + \frac{x^2}{4} + \frac{3x}{1}\right]_{-1}^{1}$

$= \left(\frac{1}{5} - \frac{1}{8} - \frac{5}{6} + \frac{1}{4} + \frac{3}{2}\right) - \left(-\frac{1}{5} - \frac{1}{8} + \frac{5}{6} + \frac{1}{4} - \frac{3}{2}\right)$

$= \frac{26}{15}$ or $1.7\dot{3}$ units^2. ✓

(2)

3.



PHI

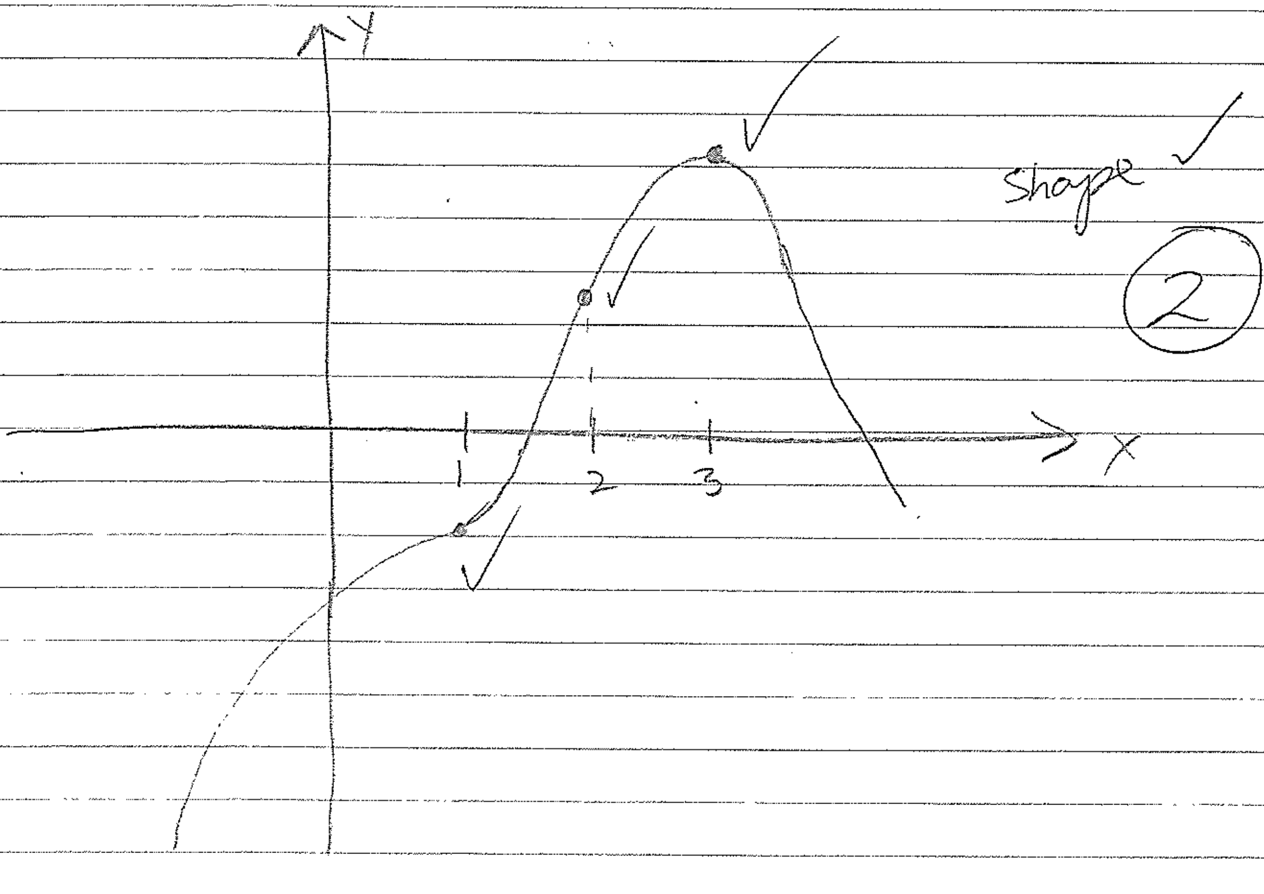
$f'(x) = 0 \Rightarrow$ turn pts at $x = 1$, $x = 3$ ← +ve, 0, −ve max t.p.

~~$f''(x) = 0 \Rightarrow$ change in concavity.~~

∴ change in concavity at $x = 2$

When $f'(x) > 0$, $f(x)$ increasing

$f'(x) < 0$, $f(x)$ decreasing



4(i) Prove $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$$\cos(2\theta+\theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$$
$$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$$
$$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$$
$$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$$
$$= 4\cos^3\theta - 3\cos\theta$$

(2)

(ii) Solve $\cos 3\theta + \sin 2\theta = \cos\theta$ for $0 \le \theta \le 2\pi$

Now $4\cos^3\theta + 2\sin\theta\cos\theta - 4\cos\theta = 0$ from (i)

$$\cos\theta(4\cos^2\theta + 2\sin\theta - 4) = 0$$
$$2\cos\theta(2\cos^2\theta + \sin\theta - 2) = 0$$
$$2\cos\theta(2(1-\sin^2\theta) + \sin\theta - 2) = 0$$
$$2\cos\theta(2 - 2\sin^2\theta + \sin\theta - 2) = 0$$
$$2\cos\theta(\sin\theta(1-2\sin\theta)) = 0$$

$\Rightarrow \cos\theta = 0$ or $\sin\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\theta = 0, \pi, 2\pi$ or $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

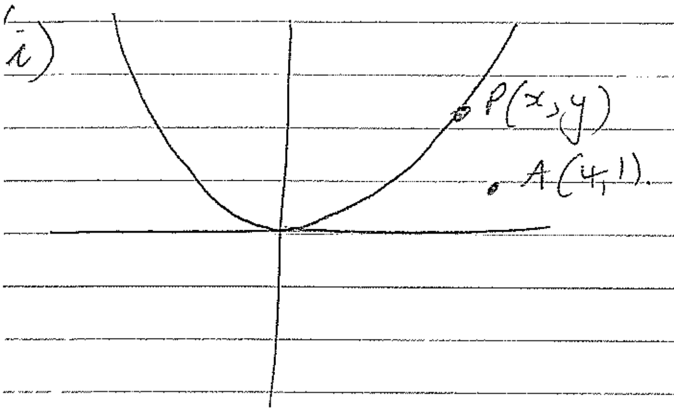
$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}, 0, \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}$

(3)

5. $x^2 = 2y$. $4a = 2$
$a = \frac{1}{2}$

$A(4,1)$ ext. pt.

(i)



$AP^2 = D^2 = (x-4)^2 + (y-1)^2$

But $y = \frac{x^2}{2}$

$\Rightarrow D^2 = (x-4)^2 + \left(\frac{x^2}{2} - 1\right)^2$ ✓ (1)

$D^2 = (x-4)^2 + \left(\frac{1}{2}x^2 - 1\right)^2$ #

(ii) $D^2 = (x-4)^2 + \left(\frac{1}{2}x^2 - 1\right)^2$

$(D^2)' = 2(x-4) + 2\left(\frac{1}{2}x^2 - 1\right)x$. ✓

$= 2x - 8 + x^3 - 2x$

$(D^2)' = x^3 - 8$

For t.p' $(D^2)' = 0 \Rightarrow x^3 = 8$
$x = \pm 2$. ✓ (3)

$(D^2)'' = 3x^2$

At $x = 2$, $(D^2)'' = 12 > 0 \Rightarrow$ min at $x = 2$.

$x = -2$, $(D^2)'' = -12 < 0 \Rightarrow$ max at $x = -2$. ✓

$\therefore$ Value of $x = 2$ to give min. for D.

(iii) $D^2 = (2-4)^2 + (2-1)^2$

$D^2 = 4 + 1$

$\Rightarrow D = \sqrt{5}$. ✓ (1)

Section C

(1)

$P(x) = (x-c)^2 Q(x)$

$P'(x) = 2(x-c)Q(x) + (x-c)^2 Q'(x)$

$P'(x) = (x-c)[2Q(x) + (x-c)Q'(x)]$

$P'(c) = (c-c)[2Q(c) + (c-c)Q'(x)]$

$P'(c) = 0$

(2)(i)

$\tan\alpha = \frac{h}{AQ} => AQ = \frac{h}{\tan\alpha}$

$\tan\beta = \frac{h}{BQ} => BQ = \frac{h}{\tan\beta}$

$L^2 = AQ^2 + BQ^2 - 2AQ \times BQ \cos 60°$

$L^2 = \frac{h^2}{\tan^2\alpha} + \frac{h^2}{\tan^2\beta} - \frac{h^2}{\tan\alpha\tan\beta}$

$L^2 = h^2\left[\frac{\tan^2\alpha + \tan^2\beta}{\tan^2\alpha\tan^2\beta} - \frac{\tan\alpha\tan\beta}{\tan^2\alpha\tan^2\beta}\right]$

$L^2 = h^2\left[\frac{(\tan\alpha + \tan\beta)^2 - 3\tan\alpha\tan\beta}{\tan^2\alpha\tan^2\beta}\right]$

$h = \frac{L\tan\alpha\tan\beta}{\sqrt{(\tan\alpha+\tan\beta)^2 - 3\tan\alpha\tan\beta}}$

(ii)

$h = \frac{xL}{\sqrt{3x + x^2 - 3x}}$

$h = L$

(3)(i)

$R\cos(\theta - \alpha) = R\cos\alpha\cos\theta + R\sin\alpha\sin\theta$

$R\cos\alpha = \sqrt{3}$

$R\sin\alpha = -1$

$R^2 = 4$

$R = 2$

$\tan\alpha = -\frac{1}{\sqrt{3}}$

$\alpha = \frac{11\pi}{6}$ or $-\frac{\pi}{6}$ 4th quadrant

$2\cos\left(\theta - \frac{11\pi}{6}\right)$ or

$2\cos\left(\theta + \frac{\pi}{6}\right)$

(ii)

$2\cos\left(\theta - \frac{11\pi}{6}\right) = 1$

$\theta - \frac{11\pi}{6} = \cos^{-1}\left(\frac{1}{2}\right)$ where $-\frac{11\pi}{6} \le \theta - \frac{11}{6} \le \frac{\pi}{6}$

$\theta - \frac{11\pi}{6} = -\frac{\pi}{3}, -\frac{5\pi}{3}$

$\theta = \frac{3\pi}{2}, \frac{\pi}{6}$

(4)(i)

$y = \frac{1}{2}(p+q)x - pq$

$0 = 2(p+q) - pq$

$pq = 2(p+q)$

(ii)

$M\left(p+q, \frac{p^2+q^2}{2}\right)$

(iii)

$x = p + q$

$y = \frac{(p+q)^2 - 2pq}{2}$

$y = \frac{x^2 - 4x}{2}$

$y = \frac{x^2}{2} - 2x$

Restrictions where $0 \le 4p - p^2$

that is $x \le 0, x \ge 8$

SECTION D

Q1 $\sin 2\theta - 3\sin^2\theta + 2\sin\theta = 0 \quad , \quad |\theta| \leq \pi$

$\therefore 2\sin\theta\cos\theta - 3\sin^2\theta + 2\sin\theta = 0$

$\sin\theta\,(2\cos\theta - 3\sin\theta + 2) = 0$

$\therefore \sin\theta = 0$ OR $2\cos\theta - 3\sin\theta + 2 = 0.$

$\theta = 0, \pm\pi$ OR $\dfrac{2(1-t^2)}{1+t^2} - \dfrac{6t}{1+t^2} + 2 = 0$

$2 - 2t^2 - 6t + 2 + 2t^2 = 0$

$6t = 4$

$t = \frac{2}{3}$

$\therefore \tan\frac{\theta}{2} = \frac{2}{3}$

$\frac{\theta}{2} = \tan^{-1}\frac{2}{3}$

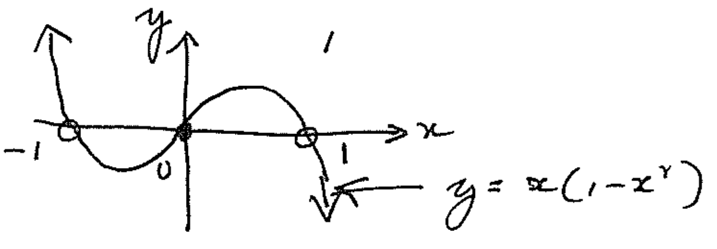
$\theta = 2\tan^{-1}\frac{2}{3}.$

$\therefore \boxed{\theta = 0, \pm\pi, 2\tan^{-1}\frac{2}{3}}$

$\sim 1.176.$

Q2. $\dfrac{x}{1-x^2} \leq 0.$

$\therefore x(1-x^2) \leq 0$ (MULTIPLY BOTH SIDES by $(1-x^2)^2$)



from graph.

$\boxed{-1 < x \leq 0, \; x > 1.}$

Q3.

now $\alpha+\beta = a+d$ —— (A)

$\alpha\beta = ad-bc$ —— (B)

now $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$= (\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)$

$= (a+d)((a+d)^2-3(ad-bc))$

$= (a+d)(a^2+2ad+d^2-3ad+3bc)$

$= a^3+2a^2d+ad^2-3a^2d+3abc$
$\quad +da^2+2ad^2+d^3-3ad^2+3bcd.$

$= a^3+d^3+3abc+3bcd.$

now required quadratic is $x^2-(\alpha^3+\beta^3)x+\alpha^3\beta^3.$

ie. $x^2-(a^3+d^3+3abc+3bcd)x+(ad-bc)^3 = 0.$

(NB. from (B) $(\alpha\beta)^3 = \alpha^3\beta^3 = (ad-bc)^3$)

Q4.

(i) $1 \times \frac{1}{3^3} + 1 \times \frac{1}{4^3} + 1 \times \frac{1}{5^3} + \cdots + 1 \times \frac{1}{n^3} = 0$

$\left[\text{OR } \sum_{r=3}^{n} \frac{1}{r^3} \right]$

(ii) now $\frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \cdots + \frac{1}{n^3} < \int_2^n x^{-3}\,dx$

$= \left[\frac{-1}{2x^2} \right]_2^n$

$= \frac{-1}{2n^2} + \frac{1}{2^3}$.

$= \frac{1}{8} - \frac{1}{2n^2}$.

$\therefore 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \cdots + \frac{1}{n^3} < \frac{1}{8} - \frac{1}{2n^2} + 1 + \frac{1}{2^3}$.

$= \frac{5}{4} - \frac{1}{2n^2}$

$\therefore$ $1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots + \frac{1}{n^3} < \frac{5}{4} - \frac{1}{2n^2}$

$< \frac{5}{4} \quad \left(\frac{1}{2n^2} > 0. \right)$